Strong decays of P wave baryons in the $1/N_c$ expansion [*]

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The hadronic decays of non-strange negative parity baryons are analyzed in the framework of the $1/N_c$ expansion. A complete basis of spin-flavor operators for the pseudoscalar meson S and D partial wave amplitudes is established to order $1/N_c$ and the unknown effective coefficients are determined by fitting to the empirically known widths. A set of relations between widths that result at the leading order, i.e. order N_c^0, is given and tested with the available data.

I. INTRODUCTION

The $1/N_c$ expansion has proven to be a very useful tool for analyzing the baryon sector. This success is mostly a consequence of the emergent contracted spin-flavor symmetry in the large N_c limit[1, 2]. For ground state baryons (identified for $N_c = 3$ with the spin 1/2 octet and spin 3/2 decuplet in the case of three flavors), that symmetry gives rise to several important relations that hold at different orders in the $1/N_c$ expansion[3–5]. The domain of excited baryons (baryon resonances) has also been explored in the framework of the $1/N_c$ expansion[6]−[14] with very promising results. The analyses carried out so far have been constrained to states that belong to a definite spin-flavor and orbital multiplet, i.e. the possibility of mixing of different such multiplets (so called configuration mixing) has been disregarded. It is very likely that such effects are small for dynamical reasons. Indeed, it has been shown that the only configuration mixings that are not suppressed by $1/N_c$ factors involve couplings to the orbital degrees of freedom[15]. The $1/N_c$ analyses of excited baryon masses have shown that orbital angular momentum couplings turn out to be very small[8, 9], which is in agreement with older results in the quark model[16]. This strongly suggests that a similar suppression, which is not a consequence of the $1/N_c$ expansion but rather of QCD dynamics, also takes place in configuration mixings. Thus, disregarding configuration mixing is likely to be a good approximation for the purpose of phenomenology. Within such a framework, a few analyses of excited baryon strong decays have been carried out, namely the decays of the negative parity SU(6) 70-plet[10] and of the Roper 56-plet[11]. In the case of interest in the present work, namely the 70-plet, the analysis in Ref. [10] used an incomplete basis of operators at sub-leading order in $1/N_c$. The main motivation of the present work (see Ref. [17] for more details) is to provide a general framework for the study of the strong decays and to perform a complete analysis to $O(1/N_c)$ in the particular case of the decays of the non-strange members of the 70-plet (i.e., the mixed symmetry 20-plet of SU(4)) into ground state baryons plus a pion or an eta meson.

This contribution is organized as follows: Section 2 contains the framework for calculating the decays, Section 3 provides the basis of effective operators, Section 4 presents the results, and finally the conclusions are given in Section 5.

II. FRAMEWORK FOR DECAYS

As discussed above in the present application of the $1/N_c$ expansion to excited baryons the assumption is made that there is an approximate spin-flavor symmetry. Thus, the excited baryons are therefore classified in multiplets of the $O(3) \times SU(2N_f)$ group. $O(3)$ corresponds to spatial rotations and $SU(2N_f)$ is the spin-flavor group where N_f is the number of flavors being considered, equal to two in the present work. The ground state baryons, namely the N and Δ states, belong to the $(\mathbf{1}, \mathbf{20_S})$ representation, where the $\mathbf{20_S}$ is the totally symmetric representation of $SU(4)$. The negative parity baryons considered here belong instead to the $(\mathbf{3}, \mathbf{20_{MS}})$ representation, where $\mathbf{20_{MS}}$ is the mixed symmetric representation of $SU(4)$. For general N_c the spin-flavor representations involve, in the Young tableaux language, N_c boxes and are identified with the totally symmetric and the mixed-symmetric representations of type $(N_c - 1, 1)$ for ground and excited negative parity states respectively. Since in the mixed symmetric spin-flavor representation one box of the Young tableaux is distinguished, such a box is associated with the "excited quark" in the baryon. In a similar fashion, and without any loss of generality it is possible to distinguish one box in the ground state multiplet as well. This is a very convenient procedure that has been used repeatedly in previous works. The

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spin and isospin quantum numbers of the distinguished box will be denoted with lower cases, and the corresponding quantum numbers of the rest of $N_c - 1$ boxes (which are in a totally symmetric representation of $SU(4)$ and form the so called "core" of the large N_c baryon) after they are coupled to eigenstates of spin and isospin will be denoted by S_c and I_c respectively. Notice that $S_c = I_c$ for totally symmetric representations of $SU(4)$. For a given core state, the coupling of the excited quark gives eigenstates of spin S and isospin I: $\mid S, S_3; I, I_3; S_c\rangle$. These states are not in an irreducible representation of $SU(4)$, as they are not in an irreducible representation of the permutation group. The totally symmetric states are given by:

$$\mid S, S_3; I, I_3\rangle_\mathbf{S} = \sum_{\eta = \pm \frac{1}{2}} C_\mathbf{S}(S, \eta) \mid S, S_3; I, I_3; S_c = S + \eta\rangle, \tag{1}$$

where

$$C_\mathbf{S}(S, \pm\frac{1}{2}) = \sqrt{\frac{(2S + 1 \pm 1)(N_c + 1 \mp (2S + 1))}{2N_c(2S + 1)}}, \tag{2}$$

while the mixed symmetric (**MS**) states $(N_c - 1, 1)$ are given by:

$$\mid S, S_3,\ I\ I_3\rangle_\mathbf{MS} = \sum_{\eta = \pm \frac{1}{2}} C_\mathbf{MS}(I, S, \eta) \mid S, S_3; I, I_3; S_c = S + \eta\rangle, \tag{3}$$

where

$$C_\mathbf{MS}(I, S, \pm\frac{1}{2}) = \begin{cases} 1 & \text{if } I = S \pm 1 \\ 0 & \text{if } I = S \mp 1 \\ \pm\sqrt{\frac{(2S+1\mp1)(N_c+1\pm(2S+1))}{2N_c(2S+1)}} & \text{if } I = S \end{cases} \tag{4}$$

Finally, upon coupling the orbital degrees of freedom, the excited baryons in the $(\mathbf{3}, \mathbf{MS})$ representation, $\mid (1, S)J, J_3; I, I_3; S\rangle_\mathbf{MS}$, are obtained.

Note that there are two sets of N^* states each consisting of two states with the same spin and isospin. The physical states are admixtures of such states, and are given by:

$$\begin{pmatrix} N_J^* \\ N_J^{*'} \end{pmatrix} = \begin{pmatrix} \cos\theta_{2J} & \sin\theta_{2J} \\ -\sin\theta_{2J} & \cos\theta_{2J} \end{pmatrix} \begin{pmatrix} {}^2N_J^* \\ {}^4N_J^* \end{pmatrix}, \tag{5}$$

where $J = \frac{1}{2}$ and $\frac{3}{2}$, $N_J^{*(')}$ are mass eigenstates, and the two mixing angles can be constrained to be in the interval $[0, \pi)$. Here the notation ${}^{2S+1}N_J^*$ has been used.

Using the standard definition for the decay width and averaging over the initial- and summing over the corresponding final-baryon spins and isospins, the decay width for each orbital angular momentum ℓ_P and isospin I_P of the pseudoscalar meson is given by

$$\Gamma^{[\ell_P, I_P]} = f_{ps} \frac{|\sum_q C_q^{[\ell_P, I_P]} B_q(\ell_P, I_P, S, I, J^*, I^*, S^*)|^2}{\sqrt{(2J^* + 1)(2I^* + 1)}}, \tag{6}$$

where the phase space factor f_{ps} is

$$f_{ps} = \frac{k_P^{1+2\ell_P}}{8\pi^2 \Lambda^{2\ell_P}} \frac{M_{B^*}}{M_B} \tag{7}$$

Here, we have used that the baryonic operator admits an expansion in $1/N_c$ and has the general form:

$$B_{[\mu, \alpha]}^{[\ell_P, I_P]} = \left(\frac{k_P}{\Lambda}\right)^{\ell_P} \sum_q C_q^{[\ell_P, I_P]}(k_P) \left(B_{[\mu, \alpha]}^{[\ell_P, I_P]}\right)_q, \tag{8}$$

where

$$\left(B_{[\mu, \alpha]}^{[\ell_P, I_P]}\right)_q = \sum_m \langle 1, m; j, j_z \mid \ell_P, \mu\rangle \, \xi_m^1 \, \left(G_{[j_z, \alpha]}^{[j, I_P]}\right)_q, \tag{9}$$

and the factor $\left(\frac{k_P}{\Lambda}\right)^{\ell_P}$ is included to take into account the chief meson momentum dependence of the partial wave. The scale Λ is chosen in what follows to be 200 MeV. Here, ξ_m^1 is an operator that produces a transition from the triplet to the singlet $O(3)$ state, and $\left(G_{[j_z,\alpha]}^{[j,I_P]}\right)_q$ is a spin-flavor operator that produces the transition from the mixed-symmetric to the symmetric $SU(4)$ representation. The label j denotes the spin of the spin-flavor operator, and as it is clear, its isospin coincides with the isospin of the emitted meson. The dynamics of the decay is encoded in the effective dimensionless coefficients $C_q^{[\ell_P,I_P]}(k_P)$. The reduced matrix elements $B_q(\ell_P, I_P, S, I, J^*, I^*, S^*)$ appearing in Eq.(6) can be easily calculated in terms of the reduced matrix elements of the spin-flavor operators. Note that in the present case, where ℓ_P can be 0 or 2 only, the spin-flavor operators can carry spin j that can be 1, 2 or 3.

The terms in the right hand side of Eq.(8) are ordered in powers of $1/N_c$. As it has been explained in earlier publications[3], the order in $1/N_c$ is determined by the spin-flavor operator. For an n-body operator, this order is given by

$$\nu = n - 1 - \kappa, \tag{10}$$

where κ is equal to zero for incoherent operators and can be equal to one or even larger for coherent operators. More details can be found in the following section where a basis of operators G is explicitly built.

With the definition of effective operators used in this work, all coefficients $C_q^{[\ell_P,I_P]}(k_P)$ in Eq.(8) are of zeroth order in N_c. The leading order of the decay amplitude is in fact N_c^0 [15]. At this point it is important to comment on the momentum dependence of the coefficients. The spin-flavor breakings in the masses, of both excited and ground state baryons, give rise to different values of the momenta k_P. In this work, we have adopted a scheme where the only momentum dependence assigned to the coefficients is the explicitly shown factor $(k_P/\Lambda)^{\ell_P}$ that takes into account the chief momentum dependence of the corresponding partial wave, and the rest of the dependence is then encoded in the coefficients of the sub-leading operators.

III. BASIS OF SPIN-FLAVOR OPERATORS

The spin-flavor operators to be considered in the present paper must connect a mixed-symmetric with a symmetric representation. Generators of the spin-flavor group acting on the states obviously do not produce such connection. However, generators restricted to act on the excited quark or on the core of $N_c - 1$ quarks can do this. The spin flavor operators can, therefore, be represented by products of generators of the spin-flavor group restricted to act either on the excited or on the core states. In the following the generators acting on the core are denoted by S_c, G_c, T_c, and the ones acting on the excited quark by s, g, t. The generators G_c are known to be coherent operators, while all the rest are incoherent. In order to build a basis of operators for the present problem one has to consider products of such generators with the appropriate couplings of spins and isospins. The n-bodyness (nB) of an operator is given by the number of such factors, and the level of coherence of the operator is determined by how many factors G_c appear in the product. It should be noticed that in the physical case where $N_c = 3$, only operators of at most 3B have to be considered. Still, to order $1/N_c$ there is a rather long list of operators of given spin j and isospin I_P. This list can be drastically shortened by applying several reduction rules. The first rule is that the product of two or more generators acting on the excited quark can always be reduced to the identity operator or to a linear combination of such generators. The second set of rules can be easily derived for products of operators whose matrix elements are taken between a mixed-symmetric and a symmetric representation. These reduction rules are as follows (here λ represents generators acting on the excited quark and Λ_c represent generators acting on the core):

$$\begin{aligned} \lambda &= -\Lambda_c \\ (\Lambda_c)_1(\Lambda_c)_2 &= -\lambda_1(\Lambda_c)_2 - \lambda_2(\Lambda_c)_1 + 1\text{B operators.} \end{aligned} \tag{11}$$

Therefore, only the following types of operators should be considered

$$\begin{array}{ccccc} \text{1B} & & \text{2B} & & \text{3B} \\ \lambda & ; & \dfrac{1}{N_c}\,\lambda_1\,(\Lambda_c)_2 & ; & \dfrac{1}{N_c^2}\,\lambda_1\,(\Lambda_c)_2\,(\Lambda_c)_3 \end{array} \tag{12}$$

It is convenient to make explicit the transformation properties of each basic operator under spin j and isospin t. In what follows we use the notation $O^{[j,t]}$ to indicate that the operator O has spin j and isospin t. It is easy to see that

$$\begin{aligned} \lambda^{[j,t]} &= s^{[1,0]},\ t^{[0,1]},\ g^{[1,1]} \\ (\Lambda_c)^{[j,t]} &= (S_c)^{[1,0]},\ (T_c)^{[0,1]},\ (G_c)^{[1,1]} \end{aligned} \tag{13}$$

For decays in the $\eta-$channels the spin-flavor operators transform as $[j,0]$ while for decays in the pion channels they should transform as $[j,1]$, where in both cases $j = 1, 2, 3$. Knowing the transformation properties of each basic operator given in Eq.(13) it is easy to construct products of the forms given in Eqs.(12) with the desired spin and isospin. Thus, the 1B operators that contribute to a given $[j,t]$ are just those $\lambda^{[j,t]}$ given in Eq.(13) which have the proper spin and isospin quantum numbers. Similarly, the possible 2B operators are given by the products $\lambda_1^{[j_1,t_1]}(\Lambda_c)_2^{[j_2,t_2]}$ coupled to the required $[j,t]$ by means of the conventional Clebsch-Gordan coefficients.

In order to construct all possible 3B operators, it is convenient to note that there are additional reduction rules for the products $(\Lambda_c)_2(\Lambda_c)_3$. The starting point is to consider all possible products of two core operators. Since one is interested in keeping contributions of at most order $1/N_c$, and because these products will appear only in 3B operators, at least one of the operators must be a G_c. Using the reduction relations[2], it can be shown[17] that the relevant list of independent products of two core operators turns out to be

$$(\{T_c, G_c\})^{[1,2]} \ , \ ([S_c, G_c])^{[1,1]} \ , \ (\{S_c, G_c\})^{[2,1]} \ , \ (G_c \ G_c)^{[2,2]} \tag{14}$$

where $([T_c, G_c])^{[1,1]}$ denotes $(T_c G_c)^{[1,1]} - (G_c T_c)^{[1,1]}$, etc.. By coupling any of these operators with one of the excited core operators $\lambda_1^{[j_1,t_1]}$ (see Eq.(13)) to the required $[j,t]$ all the possible 3B operators are obtained.

Using this scheme to couple products of generators it is straightforward to construct lists of operators with spin 1, 2 and 3 and isospin 0 and 1. Further reductions result from the fact that not all the resulting operators are linearly independent up to order $1/N_c$. The determination of the final set of independent operators for each particular decay channel is more laborious since it requires the explicitly calculation of all the relevant matrix elements. The resulting basis of independent operators $\left(O_{[m_P,I_{P3}]}^{[\ell_P,I_P]}\right)_q$ is shown in Table 1, where for simplicity the corresponding spin and isospin projections have been omitted. Finally, the normalized basis operators $\left(B_{[m_P,I_{P3}]}^{[\ell_P,I_P]}\right)_q$ are defined. They differ from those listed in Table 1 by a normalization constant which is determined by requiring that, for $N_c = 3$, their largest reduced matrix element should be equal to one for order N_c^0 operators and equal to $1/3$ for order $1/N_c$ operators.

TABLE I: Operator basis

	n-bodyness	Name	Operator	Order in $1/N_c$
	1B	$O_1^{[0,1]}$	$(\xi \ g)^{[0,1]}$	0
Pion		$O_2^{[0,1]}$	$\frac{1}{N_c} \left(\xi \left(s \ T_c\right)^{[1,1]}\right)_{[0,a]}^{[0,1]}$	1
S wave	2B	$O_3^{[0,1]}$	$\frac{1}{N_c} \left(\xi \left(t \ S_c\right)^{[1,1]}\right)_{[0,a]}^{[0,1]}$	1
		$O_4^{[0,1]}$	$\frac{1}{N_c} \left(\xi \left(g \ S_c\right)^{[1,1]}\right)_{[0,a]}^{[0,1]}$	1
	1B	$O_1^{[2,1]}$	$(\xi \ g)_{[i,a]}^{[2,1]}$	0
		$O_2^{[2,1]}$	$\frac{1}{N_c} \left(\xi \left(s \ T_c\right)^{[1,1]}\right)_{[i,a]}^{[2,1]}$	1
Pion		$O_3^{[2,1]}$	$\frac{1}{N_c} \left(\xi \left(t \ S_c\right)^{[1,1]}\right)_{[i,a]}^{[2,1]}$	1
D wave	2B	$O_4^{[2,1]}$	$\frac{1}{N_c} \left(\xi \left(g \ S_c\right)^{[1,1]}\right)_{[i,a]}^{[2,1]}$	1
		$O_5^{[2,1]}$	$\frac{1}{N_c} \left(\xi \left(g \ S_c\right)^{[2,1]}\right)_{[i,a]}^{[2,1]}$	1
		$O_6^{[2,1]}$	$\frac{1}{N_c} \left(\xi \left(s \ G_c\right)^{[2,1]}\right)_{[i,a]}^{[2,1]}$	0
	3B	$O_7^{[2,1]}$	$\frac{1}{N_c^2} \left(\xi \left(s \left(\{S_c, G_c\}\right)^{[2,1]}\right)^{[2,1]}\right)_{[i,a]}^{[2,1]}$	1
		$O_8^{[2,1]}$	$\frac{1}{N_c^2} \left(\xi \left(s \left(\{S_c, G_c\}\right)^{[2,1]}\right)^{[3,1]}\right)_{[i,a]}^{[2,1]}$	1
Eta	1B	$O_1^{[0,0]}$	$(\xi \ s)_{[0,0]}^{[0,0]}$	0
S wave	2B	$O_2^{[0,0]}$	$\frac{1}{N_c} \left(\xi \left(s \ S_c\right)^{[1,0]}\right)_{[0,0]}^{[0,0]}$	1
Eta	1B	$O_1^{[2,0]}$	$(\xi \ s)_{[i,0]}^{[2,0]}$	0
D wave	2B	$O_2^{[2,0]}$	$\frac{1}{N_c} \left(\xi \left(s \ S_c\right)^{[1,0]}\right)_{[i,0]}^{[2,0]}$	1
		$O_3^{[2,0]}$	$\frac{1}{N_c} \left(\xi \left(s \ S_c\right)^{[2,0]}\right)_{[i,0]}^{[2,0]}$	1

IV. RESULTS

The different empirical S- and D-wave decay widths used in the analysis are the ones provided by the PDG[18]. The values for the widths and branching ratios are taken as the ones indicated there as "our estimate", while the errors are determined from the corresponding ranges. The corresponding partial widths obtained from those values are explicitly displayed in Table 3. The entries indicated as unknown reflect channels for which no width is provided by the PDG or where the authors consider that the input is unreliable, such as in the $\pi\Delta$ decay modes of the $N(1700)$ and $N(1675)$ and the D-wave $\eta-$ decay modes. At this point it is important to stress the marginal precision of the data for the purposes of this work. This work performs an analysis at order $1/N_c$, which means that the theoretical error is order $1/N_c^2$. This implies that amplitudes are affected by a theoretical uncertainty at the level of 10%. Thus, in order to pin down the coefficients of the subleading operators, the widths provided by the data should not be affected by errors larger than about 20%. As shown in Table 3, the experimental errors are in most entries 30% or larger. In consequence, the determination of the subleading effective coefficients is affected by large errors as the results below show.

Before presenting the results of the fits, it is convenient to derive some parameter independent relations that can be obtained to leading order. These relations serve as a test of the leading order approximation. Since at this order there are only four coefficients and two angles to be fitted, and there are a total of twenty partial widths (excluding all D-wave η channels but including kinematically forbidden η-channel decays), there are fourteen independent parameter free relations that can be derived. These relations are more conveniently written in terms of reduced widths, i.e., widths where the phase space factor f_{sp} (see Eq.(7)) has been removed and denoted here by $\tilde{\Gamma}$. Considering the S-wave decays in the π mode, there are six decays and three parameters in the fit. Thus, three parameter free relations must follow. These relations and the corresponding comparison with experimental values read:

$$
\begin{array}{ccccccc}
 & \begin{array}{c} \tilde{\Gamma}_{N(1535)\to\pi N} \\ +\tilde{\Gamma}_{N(1650)\to\pi N} \end{array} & : & \begin{array}{c} \tilde{\Gamma}_{N(1520)\to\pi\Delta} \\ +\tilde{\Gamma}_{N(1700)\to\pi\Delta} \end{array} & : & \tilde{\Gamma}_{\Delta(1620)\to\pi N} & : & \tilde{\Gamma}_{\Delta(1700)\to\pi\Delta} \\
\text{Th.} & 1 & : & 1 & : & 0.17 & : & 0.42 \\
\text{Exp.} & 1 & : & \text{unknown} & : & 0.19\pm0.07 & : & 0.62\pm0.33
\end{array}
\tag{15}
$$

Within the experimental errors the relations are satisfied. In a similar fashion, relations involving D- wave decays in the π mode can be obtained. There are in this case four parameters to fit and eleven partial widths. Thus, seven parameter free relations can be obtained. In general these relations are quadratic and/or involve some of the unknown decay widths. However, three testable linear relations can be obtained. The simplest of them reads,

$$
\begin{array}{cccc}
2\,\tilde{\Gamma}_{\Delta(1620)\to\pi\Delta} + \tilde{\Gamma}_{\Delta(1700)\to\pi\Delta} & = & 8\,\tilde{\Gamma}_{\Delta(1700)\to\pi N} + \dfrac{15}{4}\,\tilde{\Gamma}_{N(1675)\to\pi N} \\
\text{Exp.} \quad 5.24\pm1.95 & = & 2.19\pm0.62
\end{array}
$$

As we see this relation is not well satisfied by the empirical data. Something similar happens with the other two. Thus, one can anticipate a poor leading order fit to the D-wave pion decays.

In Table 2 the results of several fits are displayed. In these fits the decay amplitudes are expanded keeping only the terms that correspond to the order in $1/N_c$ of the fit. Similarly, when performing the fits the errors have been taken to be equal or larger than the expected accuracy of the fit (30% to the LO fits and about 10% for the NLO ones).

The first LO fit only considers the S-wave π-modes. One notices here that both θ_1 and θ_3 have a two fold ambiguity at this order. The second leading order fit includes the D-waves and η-modes. The angles remain within errors equal to the ones from the first fit. Table 3 shows that the $N(1535)\to\eta N$ width results to be a factor four smaller than the empirical one (this having, however, a rather generous error). In the D-waves, several widths involving decays with a Δ in the final state are also too small. The S-wave π-modes are well fitted and there is no real need for NLO improvement. In the D-wave decays in the π channel there are two leading order operators that contribute. The fit shows that the 1B operator has a coefficient whose magnitude is a factor two to three larger than that of the coefficient of the 2B operator. The 1B D-wave operator as well as the 1B S-wave operator $O_1^{[0,1]}$ stem from the 1B coupling of the pion via the axial current. Such a coupling naturally occurs as the dominant coupling in the chiral quark model[19].

The NLO fit involves a rather large number of effective constants. Since there are only sixteen data available, some operators must be discarded for the purpose of the fit. It is reasonable to choose to neglect the 3B operators and the subleading S-wave operator for η emission. The NLO fit has been carried out by demanding that the LO coefficients are not vastly different from their values obtained in the LO fit. This demand is reasonable if the assumption is made that the $1/N_c$ expansion makes sense. The fact that the NLO coefficients do not have unnaturally large values with respect to the scale set by the LO fit indicates the consistency of the assumption. This clearly is no proof, however,

TABLE II: Fit parameters. Fit #1: Pion S-waves LO. In this case there is a four-fold ambiguity for the angles $\{\theta_1, \theta_3\}$ given by the two values shown for each angle. Fit #2: Pion S and D-waves, eta S-waves, LO. In this case there is a two-fold ambiguity for the angle θ_1. For the angle θ_3 there is an *almost* two-fold ambiguity given by the two values indicated in parenthesis and which only differ in the two slightly different values of $C_6^{[2,1]}$. Fit #3: Pion S and D-waves, eta S-waves, NLO, no 3-body operators. No degeneracy in θ_1 but *almost* two-fold ambiguity in θ_3 given by the two values indicated in parenthesis. Values of coefficients which differ in the corresponding fits are indicated in parenthesis.

Coefficient	#1 LO	#2 LO	#3 NLO
$C_1^{[0,1]}$	31 ± 3	31 ± 3	23 ± 3
$C_2^{[0,1]}$	-	-	$(7.4, 32.5) \pm (27, 41)$
$C_3^{[0,1]}$	-	-	$(20.7, 26.8) \pm (12, 14)$
$C_4^{[0,1]}$	-	-	$(-26.3, -66.8) \pm (39, 65)$
$C_1^{[2,1]}$	-	4.6 ± 0.5	3.4 ± 0.3
$C_2^{[2,1]}$	-	-	-4.5 ± 2.4
$C_3^{[2,1]}$	-	-	$(-0.01, 0.08) \pm 2$
$C_4^{[2,1]}$	-	-	5.7 ± 4.0
$C_5^{[2,1]}$	-	-	3.0 ± 2.2
$C_6^{[2,1]}$	-	$(-1.86, -2.25) \pm 0.4$	-1.73 ± 0.26
$C_7^{[2,1]}$	-	-	-
$C_8^{[2,1]}$	-	-	-
$C_1^{[0,0]}$	-	11 ± 4	17 ± 4
$C_2^{[0,0]}$	-	-	-
θ_1	1.62 ± 0.12 0.29 ± 0.11	1.56 ± 0.15 0.35 ± 0.14	0.39 ± 0.11
θ_3	3.01 ± 0.07 2.44 ± 0.06	$(3.00, 2.44) \pm 0.07$	$(2.82, 2.38) \pm 0.11$
χ^2_{dof}	0.25	1.5	0.9
dof	2	10	3

TABLE III: Partial widths resulting from the different fits in Table II. Values indicated in parenthesis correspond to the cases in which *almost* degenerate values of θ_3 lead to different partial widths.

	Decay	Emp. [MeV]	#1 LO [MeV]	#2 LO [MeV]	#3 NLO [MeV]
	$N(1535) \to \pi N$	68 ± 27	74	62	(58,68)
	$N(1520) \to \pi \Delta$	10 ± 4	10	9.7	9.5
π	$N(1650) \to \pi N$	121 ± 40	132	144	122
S-wave	$N(1700) \to \pi \Delta$	unknown	175	175	(259,156)
	$\Delta(1620) \to \pi N$	38 ± 13	35	35	38
	$\Delta(1700) \to \pi \Delta$	112 ± 53	81	81	(135,112)
	$N(1535) \to \pi \Delta$	1 ± 1		.01	0.5
	$N(1520) \to \pi N$	67 ± 9		70	65
	$N(1520) \to \pi \Delta$	15 ± 3		2.8	13
	$N(1650) \to \pi \Delta$	7 ± 5		0.12	8
π	$N(1700) \to \pi N$	10 ± 7		10	(11,9)
D-wave	$N(1700) \to \pi \Delta$	unknown		4	(4,9)
	$N(1675) \to \pi N$	72 ± 12		85	76
	$N(1675) \to \pi \Delta$	unknown		45	79
	$\Delta(1620) \to \pi \Delta$	68 ± 26		30	87
	$\Delta(1700) \to \pi N$	45 ± 21		49	32
	$\Delta(1700) \to \pi \Delta$	12 ± 10		15	18
η	$N(1535) \to \eta N$	64 ± 28		17	(57,61)
S-wave	$N(1650) \to \eta N$	11 ± 6		14	12

that the $1/N_c$ expansion is working. As mentioned earlier, the chief limitation here is due to the magnitude of the errors in the inputs. This leads to results for the NLO coefficients being affected with rather large uncertainties. Indeed, no clear NLO effects can be pinned down, as most NLO coefficients are no more than one standard deviation from zero. Because the number of coefficients is approximately equal to the number of inputs, there are important correlations between them. For instance, the S-wave NLO coefficients are very correlated with each other and with the angle θ_3. For the S-waves the LO fit is already excellent, and therefore nothing significantly new is obtained by including the NLO corrections. Correlations are smaller for the D-wave coefficients. As mentioned before, here the LO fit has room for improvement, and thus the NLO results are more significant than in the case of the S-waves. Still, no clear pattern concerning the NLO corrections is observed. One interesting point, however, is that without any significant change in the value of θ_1 the η-modes are now well described. The reason for this is that in the LO fit the matrix elements of the operator $O_1^{[0,0]}$ were taken to zeroth order in $1/N_c$, while in the NLO fit the $1/N_c$ terms are included. These subleading corrections enhance the amplitude for the $^2N_{1/2}^*$ and suppress the amplitude for the $^4N_{1/2}^*$. This along with an increment in the coefficient brings the fit in line with the empirical widths. One important point is that at NLO the two fold ambiguity in θ_1 that results at LO is eliminated. The smaller mixing angle turns out to be selected. The angle θ_3 remains ambiguous and close to the values obtained in the LO fits. It should be noticed that the present values of both mixing angles are somewhat different from the values $\theta_1 = 0.61$, $\theta_3 = 3.04$ obtained in other analyses[10, 16, 20].

V.   CONCLUSIONS

In this work we have analyzed the strong decays of the negative parity excited baryons in the $1/N_c$ expansion assuming that configuration mixings can be neglected. A basis of effective operators, in which the S- and D-wave amplitudes are expanded, was constructed to order $1/N_c$. All dynamical effects are then encoded in the effective coefficients that enter in that expansion. We find that a consistent description of the decays within the $1/N_c$ expansion is possible. Indeed, up to the relatively poor determination of $1/N_c$ corrections that results from the magnitude of the errors in the input widths, these corrections are of natural size. A few clear cut observations can be made. The most important one is that the S-wave π- and η-channels are well described by the leading order operators (one for each channel) provided one includes the contributions subleading in $1/N_c$ in the matrix elements for the η-decays. The mixing angle θ_1 is then determined by these channels up to a twofold ambiguity, which is lifted when all channels are analyzed at NLO. The angle θ_3 is also determined up to a two fold ambiguity at LO. The ambiguity remains when the NLO is considered. The subleading operators are shown to be relevant to fine tune the S-wave decays and improve the D-wave decays. Because of the rather large error bars in and significant correlations between the resulting effective coefficients, no clear conclusions about the physics driving the $1/N_c$ corrections can be made. The mixing angles $\theta_1 = 0.39 \pm 0.11$ and $\theta_3 = (2.82, 2.38) \pm 0.11$ that result at NLO are similar to the ones determined at LO. They are , however, somewhat different from the angles $\theta_1 = 0.61$ and $\theta_3 = 3.04$ obtained in other analyses[10, 16, 20].

Acknowledgments

[1] J. L. Gervais and B. Sakita, *Phys. Rev. Lett.* **52**, 87 (1984); *Phys. Rev. D* **30**, 1795 (1984).
[2] R. Dashen and A. V. Manohar, *Phys. Lett. B* **315**, 425 (1993); *Phys. Lett. B* **315**, 438 (1993); E. Jenkins, *Phys. Lett. B* **315**, 441 (1993).
[3] R. Dashen, E. Jenkins, and A. V. Manohar, *Phys. Rev. D* **49**, 4713 (1994); R. Dashen, E. Jenkins, and A. V. Manohar, *Phys. Rev. D* **51**, 3697 (1995).
[4] C. D. Carone, H. Georgi and S. Osofsky, *Phys. Lett. B* **322**, 227 (1994); M. A. Luty and J. March-Russell, *Nucl. Phys. B* **426**, 71 (1994); M. A. Luty, J. March-Russell and M. White, *Phys. Rev. D* **51**, 2332 (1995).
[5] E. Jenkins and R. F. Lebed, *Phys. Rev. D* **52**, 282 (1995); J. Dai, R. Dashen, E. Jenkins, and A. V. Manohar, *Phys. Rev. D* **53**, 273 (1996); A. Buchmann and R. F. Lebed, *Phys. Rev. D* **62**, 096005(2000); A. Buchmann, J. A. Hester and

R. F. Lebed, *Phys. Rev.* D **66**, 056002(2002); E. Jenkins, X. d. Ji and A. V. Manohar, *Phys. Rev. Lett.* **89**, 242001 (2002); T. D. Cohen, *Phys. Lett.* B **554**, 28 (2003).

[6] J. L. Goity, *Phys. Lett.* B **414**, 140 (1997).

[7] D. Pirjol and T. M. Yan, *Phys. Rev.* D **57**, 1449 (1998); *Phys. Rev.* D **57**, 5434 (1998).

[8] C. E. Carlson, C. D. Carone, J. L. Goity and R. F. Lebed, *Phys. Lett.* B **438**, 327 (1998); *Phys. Rev.* D **59**, 114008 (1999).

[9] C. L. Schat, J. L. Goity and N. N. Scoccola, *Phys. Rev. Lett.* **88**, 102002 (2002); *Phys.Rev.* D **66**, 114014 (2002); *Phys. Lett.* B **564**, 83 (2003).

[10] C. D. Carone, H. Georgi, L. Kaplan and D. Morin, *Phys. Rev.* D **50**, 5793 (1994).

[11] C. E. Carlson and C. D. Carone, *Phys. Lett.* B **484**, 260 (2000).

[12] N. Matagne and Fl. Stancu, *Phys. Rev.* D **71**, 014010 (2005).

[13] T. D. Cohen and R. F. Lebed, *Phys. Rev. Lett.* **91**, 012001 (2003); *Phys. Rev.* D **67**, 096008 (2003); *Phys. Rev.* D **70**, 096015 (2004); T. D. Cohen, D. C. Dakin, A. Nellore and R. F. Lebed, *Phys. Rev.* D **69**, 056001 (2004); *Phys. Rev.* D **70**, 056004 (2004).

[14] D. Pirjol and C. Schat, *Phys. Rev.* D **67**, 096009 (2003); *AIP Conf. Proc.* **698**, 548 (2004).

[15] J. L. Goity, *Yad. Fiz.* **68**, 655 (2005) [*hep-ph/0405304*]; contribution to these proceedings.

[16] S. Capstick and W. Roberts, *Prog. Part. Nucl. Phys.* **45**, S241 (2000) and references therein.

[17] J. L. Goity, C. Schat and N. N. Scoccola, *Phys. Rev.* D **71**, 034016 (2005).

[18] Particle Data Group, S. Eidelman et al., *Phys. Lett.* B **592**, 1 (2004).

[19] A. V. Manohar and H. Georgi, *Nucl. Phys.* B **234**, 189 (1984); L. Y. Glozman and D. O. Riska, *Phys. Rept.* **268**, 263 (1996).

[20] D. Faiman and D. E. Plane, *Nucl. Phys.* B **50**, 379 (1972); A. J. G. Hey, P. J. Litchfield and R. J. Cashmore, *Nucl. Phys.* B **95**, 516 (1975).